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## TRIP CHAINING: WHO WINS WHO LOSES?

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# TRIP CHAINING: WHO WINS WHO LOSES?<sup>1</sup>

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**Abstract:** This paper studies how trip chaining (combining commuting and shopping or commuting and child care) affects market competition: in particular, pricing and the equilibrium number of firms as well as welfare. We use a monopolistic competition framework, where firms sell differentiated products as well as offering differentiated jobs to households, who are all located at some distance from the firms. The symmetric equilibria with and without the option of trip chaining are compared. We show analytically that introducing the trip chaining option reduces the profit margin of the firms in the short run, but increases welfare. The welfare gains are, however, smaller than the transport cost savings. In the free-entry long run equilibrium, the number of firms decreases but welfare is higher. A numerical illustration gives orders of magnitude of the different effects.

**Keywords:** trip chaining, discrete choice model, imperfect competition, wage and price competition.

**JEL classification:** D43, L13, R3

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**Abstract**

This paper studies how trip chaining (combining commuting and shopping or commuting and child care) affects market competition: in particular, pricing and the equilibrium number of firms as well as welfare. We use a monopolistic competition framework, where firms sell differentiated products as well as offering differentiated jobs to households, who are all located at some distance from the firms. The symmetric equilibria with and without the option of trip chaining are compared. We show analytically that introducing the trip chaining option reduces the profit margin of the firms in the short run, but increases welfare. The welfare gains are, however, smaller than the transport cost savings. In the free-entry long run equilibrium, the number of firms decreases but welfare is higher. A numerical illustration gives orders of magnitude of the different effects.

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## 1. INTRODUCTION

According to Hensher and Reyes (2000) "...individuals increasingly seek out opportunities to minimise the amount of travel as part of activity fulfillment, given the competing demands on time budgets and their valuation of travel time savings. This search for ways of fulfilling (more) activities with less travel input has produced a number of responses, one of which is trip chaining". Recent empirical evidence supports this statement. For instance, relying on data from the 1995 Nationwide Personal Transportation Survey and the 2001 National Household Travel Survey to examine trip chaining trends in the United States, McGuckin, Zmud and Nakamoto (2005) establish that a robust growth in trip chaining occurred between 1995 and 2001, especially in the direction from home to the workplace and for shopping purposes. We define trip chaining as a stop for shopping on the home-to-work or work-to-home journey. Our insights on trip chaining extend beyond the commuting –shopping combination and could also apply to the choice faced by a couple combining work and school trips or combining work and day care trips. Allowing trip chaining requires building in flexibility in terms of shop (school, day care etc.) hours and work hours, parking and public transit facilities. Practices differ strongly among regions: compare the rigid North of Germany (fixed working hours, limited day care and shopping) with the more flexible Belgium (working hours, shopping and day care facilities more flexible). This often requires the coordinated action of firms and labor unions as well as governments. Agreement will depend on the welfare effects that can be expected from offering the trip chaining options. We show how firms, consumers and workers are affected by the trip chaining option. Among the important insights are that the savings in transport costs that accompany trip chaining options are not a correct estimate of the welfare effects and that the profit, the number and diversity of firms will decrease but that welfare will increase.

Most trip chaining research has concentrated on the product side and has looked into the effect of trip chaining on the pricing of products. Raith (1996) takes the commuting trip as given so that making the trip chaining option available increases the competition among shops along the commuting route. In this paper we pursue a different avenue of research. We take a general equilibrium approach, where every subcenter at which one shops is also a differentiated workplace. Individuals choose a workplace and may or may not combine this with shopping.

This means that product prices and wages are determined simultaneously. The general equilibrium (GE) approach plays an essential role in our model. Without the GE approach, one implicitly assumes that the product (or labor) market has perfect competition. Whenever the product market is perfectly competitive, it does not matter where an individual shops as all places offer the same product. The individual then always shops where he works: there is always 100% trip chaining whenever it is allowed. This is clearly not a satisfactory approach as this level of trip chaining is not observed in practice. So, in order to explain the work-shop or work-day care choices that are observed, we need to study the diversity of work and shopping opportunities simultaneously.

The research questions we address are therefore: does the introduction of trip chaining possibilities increase or decrease the profit margins, does this in the end also lead to more or fewer firms and how does it affect welfare? This requires the careful analysis of two forces. On the one hand, a worker may be ready to pay more when he patronizes the shopping center where he works and this tends to increase the spatial monopoly power of shops with trip chaining. On the other hand, trip chaining reduces the average distance between consumers and shops and this tends to increase competition in the standard spatial model à la Hotelling.

Our starting point is an analytical, symmetric model of a city, in which households live in the city center and shop and work in equidistant subcenters (de Palma and Proost, 2006). Each subcenter offers a different variety of the product and offers a different type of workplace. A unique firm located in each subcenter maximizes profits by setting a price and a wage that attracts the optimal number of customers and attracts the necessary workers to supply the demand addressed to it. In the de Palma and Proost (2006) model, there exists a unique symmetric short term Nash equilibrium in prices and wages and a free entry equilibrium. In the original model individuals made separate working and shopping trips. In this paper we relax this assumption and allow consumers to shop at the subcenter where they work (trip-chain).

For this model, we show that a symmetric short term and a free entry Nash equilibrium exist when the trip chaining option is introduced. We present four key results. Firstly, in the short run, trip chaining will increase competition between subcenters and decrease mark ups as long as the preference for variety in the product space is strictly smaller than the preference for variety

in the workplace space. Secondly, allowing trip chaining benefits consumers and increases welfare but the gain is smaller than the savings in transport costs. Thirdly, in the free entry equilibrium, the trip chaining option decreases the number of firms. Finally, the welfare of the free entry equilibrium is higher with the trip chaining option than without the trip chaining option.

In a first step we use a model in which the location of shops and households is fixed. An important extension would be to introduce the endogenous formation and location of shopping centers along the lines of Fujita and Thisse (2002). The current paper is theoretical and complements the research on trip chaining, which is mostly based on simulation (agent-based models) or on empirical analysis (see Bhat et al, 2004a and 2004b, and Bhat, 2008, for example).

We start in Section 2 with a brief comparison of our city lay-out with alternative city layouts. This is important as the city lay-out will determine the role of transport costs in the competition between shops and workplaces. The model is described in Section 3 and the short-run and free entry equilibria with trip chaining and without trip chaining are compared. In Section 4 we look at the welfare implications of trip chaining. A small numerical illustration to show the relative importance of different parameters is included in Section 5 and Section 6 concludes.

## 2. TRIP CHAINING IN DIFFERENT SPATIAL STRUCTURES

Trip chaining can be studied in many different spatial structures. Each structure generates its own results. We use a highly stylized spatial structure that offers maximum symmetry and consistency. In this section we compare our spatial structure with a few alternatives.

**(Figure 1 about here)**

The city layout we use in this paper is represented in Figure 1. There is one city center; people live there but work and shop in subcenters around the city. Every subcenter offers a differentiated product and a differentiated job. When individuals cannot combine shopping and commuting trips, they have to make two trips, where they select the best shopping variant and the best job variant. When individuals can combine work and shopping trips, they trade off the transport cost with the loss of variety they experience when they have to shop at the workplace

location. The city layout is completed by adding intermediate goods that are transported from the city center to the subcenters. The advantage of the chosen layout is the symmetry: the number of subcenters is fully variable and combining job and shopping always saves the same transport cost (from center to subcenter).

**(Figure 2 about here )**

In the triangular structure of Figure 2 one can imagine that initially the same number of people live in every center. If combining shopping and work trips is not allowed, they can choose to work at home or in one of the three other centers and can also choose to shop in one of the three centers. The asymmetry between working and shopping at home and in another center means that every firm enjoys some “transport cost” protection at home for the product and labor market. The same types of forces will be at work as in our model set-up. There will be two differences: firstly, the asymmetry between shopping or working at home and in another subcenter makes results less transparent; and secondly, one cannot study the free entry of new centers as the triangular design does not allow this to be done in a symmetric way. .

**( Figure 3 about here )**

Still another layout is Figure 3 that represents a circle where individuals are either located in one of the centers or are distributed uniformly along a ring road. They have to choose a center for working and shopping. Every center again has some transport cost protection for individuals located nearby. One can study competition between the centers for shoppers and workers. Whenever trip chaining is allowed there is an advantage to shop on one’s trip to work if this does not imply giving up too much variety. This set up allows any number of centers (products and jobs) but, again, studying competition among centers is less transparent, as there is an asymmetry in the competition for individuals generated by the difference in transport costs.

**(Figure 4 about here)**

The circular layout of Figure 3 is often used for studying product variety but a simple layout along one line as in Figure 4 may be more realistic to study transport effects. Individuals are located in centers or uniformly on the line. In the absence of trip combination options, every center again enjoys transport protection when it competes for customers or workers. As before,



making the option of trip chaining available generates the trade-off between giving up diversity and transport costs. The layout of Figure 4 has been used by Claycombe (1991), Claycombe and Mahan (1993) and Raith (1996). In their approach the workplace and the wage are fixed and they concentrate on the shopping market only. The shopping market is represented by a Hotelling type of model where evenly spaced shops offering identical products are placed along one infinite line. Evenly distributed consumers have to make exogenously determined commuting trips of a given distance and may stop for shopping on their commuting trip. Raith proves that an increasing commuting distance means that more shops will be encountered on the trip to work and this implies more intensive price competition. Our approach is different on three counts. Firstly, we use a monopolistic competition model with differentiated products which allows the effect of preference for variety to be studied. Secondly, we model the two trip purposes simultaneously, as we have a general equilibrium model with differentiated workplaces and differentiated products. This is important as both markets interact: a shop that sells more needs to attract more workers. Lastly, the number of firms can be endogenous in our approach as we also study the free entry equilibrium. Comparing our city lay-out with the linear lay-out we have the advantage of full symmetry in the competition between subcenters.

This section has shown that the city lay-out matters for studying trip chaining. The framework we select offers full symmetry between competitors and new features like the simultaneous competition on product and labor markets can therefore be more easily integrated into the study of trip chaining.

### 3. THEORETICAL FRAMEWORK

#### 3.1. *The Framework*

Imperfect competition in a city both with and without congestion has been analysed recently for a closed economy by de Palma and Proost (2006) using the city layout of Figure 1. In their model, all households live in the city center and make trips to work and shop at  $n$  subcenters ( $n \geq 2$ ) that are located at identical travel cost from the city center. In each subcenter there is one firm that offers a differentiated product and a differentiated work place. The firms compete

in wages on the labor market to attract workers and compete in prices to attract customers. Households are constrained to make separate trips for shopping and working, so trip-chaining is *de facto* not permitted. In the current paper we relax this assumption and allow residents to shop at their work location without making a separate journey. The model set-up is still symmetric but, in contrast to de Palma and Proost (2006), we do not include congestion in order to focus solely on the effect trip-chaining has on the equilibrium. In this section we provide a brief description of the model set-up and derive the relevant expressions for the symmetric price equilibrium without congestion but with trip chaining.

All trips are between the city center and the subcenters. Residents cannot travel directly from one subcenter to another but are allowed to combine a working and a shopping trip. Residents first choose where to work and then decide whether to shop at their work location or at another subcenter; however residents can only travel between the center and each subcenter and not between subcenters.

A homogeneous good is produced in the city center and used as an intermediate input for the differentiated good produced in the subcenters. A quantity  $c$  of homogeneous good is necessary per unit of the differentiated good. Each of the  $N$  households supplies  $\theta$  units of homogeneous labor for the production of the homogeneous good in the city center. Each household also buys exactly one unit of the differentiated good and supplies exactly one unit of differentiated labor for the production of differentiated goods in the subcenter. Summing up, in order to produce its variety of the differentiated good, each of the  $n$  sub centers needs four inputs: the intermediate inputs ( $c$  per unit), one unit of differentiated labor, a fixed set up cost ( $F$  units of the homogeneous good) and a public capital good (roads, parking etc.) that requires  $K$  units of the homogeneous good.

The total production possibilities of an economy with  $N$  households and  $n$  firms can then be expressed in terms of the following identity for total labor supply and demand:

$$(1 + \theta)N = D + cD + nF + nK + \left[ \alpha^w + (1 - \delta)\alpha^d + \alpha^h \right] tD + G. \quad (1)$$

The left hand side represents the total labor supply that is fixed: for each household we have one unit of labor supplied to the subcenters and  $\theta$  units of labor supplied to the production of homogeneous goods. The total demand for the differentiated good is given by  $D$ .

Both firms and households incur travel costs. Households have to make trips from the center to the subcenters for working and shopping and firms have to bring the intermediate input from the center to the subcenter. We assume that the transport cost per trip is  $t$  (measured in units of homogenous labor foregone per trip). The total transportation costs for commuting, shopping and shipping goods to the sub centers are given by  $[\alpha^w + (1-\delta)\alpha^d + \alpha^h]tD$ . The parameters  $\alpha^w, \alpha^d$  and  $\alpha^h$  represent, respectively, the number of commuting and shopping trips the consumer<sup>2</sup> undertakes per unit of production (respectively consumption) of the differentiated good and the number of shipping trips that are necessary to deliver the intermediate good to the subcenter. We assume that the frequency of shopping trips is less than or equal to the commuting frequency ( $\alpha^d \leq \alpha^w$ ). The variable  $\delta (\leq 1)$  effectively represents the fraction of consumers who take advantage of trip-chaining. When trip chaining is not possible each unit of the differentiated good that is bought requires  $\alpha^d tD$  in terms of transport costs and  $\delta$  equals zero. When trip chaining is an option,  $\delta$  is endogenous and transport costs for shopping can be lower and are equal to  $(1-\delta)\alpha^d tD$ .

The frequency of shopping  $\alpha^d$  (once a week or every day) is important because it determines the transport cost savings of trip chaining: when every day shopping requirements (fresh food, etc) can be combined with a commuting trip, this represents a more important saving than the weekly grocery shopping trip. For this reason, the term  $\alpha^d t$  will appear in the expressions that determine the advantage of trip chaining for the consumers.

The last term in (1),  $G$ , represents the total quantity of the homogenous consumption good that will be available after all other production costs and transport costs related to the differentiated goods have been accounted for.

Inspection of equation (1) can give us a flavor of the trade-offs involved in trip chaining. Firstly, when the proportion of trip chaining  $\delta$  increases, transport costs are reduced and this allows higher consumption possibilities. Secondly, trip chaining also means that some consumers may

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<sup>2</sup> In the following we will use household and consumer interchangeably as it is easier to consider the household as a single worker or shopper.

give up their preferred product variant in order to save on transport costs. The latter implies that welfare gains may be smaller than the transport costs saved. Finally, trip chaining may, by affecting the profit margins, also affect the number of subcenters in equilibrium and affect welfare in the long term. A smaller number of subcenters saves on fixed costs but leads to a loss of diversity that itself has welfare cost. So the number of households that trip chains ( $\delta$ ) is endogenous and is the result of the interplay of household decisions and firms' pricing and entry decisions.

As the household preference for variety plays a key role in the trip chaining process, we first define the specification of the working and shopping preferences of the households. Next we address the behavior of the firm and we conclude with an analysis of the market equilibrium.

In order to make the model complete, we define the government budget equilibrium and the ownership of the firms. The only role of government in this model is to supply the fixed public inputs ( $K$  per subcenter) and to finance this supply via a head tax on households  $T$  and a fixed levy  $S$  per firm. The government budget equilibrium requires  $nK = nS + NT$ . The ownership of all firms and their net profits are divided evenly between the  $N$  individuals.<sup>3</sup>

### 3.2. *Households' Behavior*

Household utility is represented by a linear function of the utility obtained from consumption of the differentiated and homogeneous goods and the disutility of supplying labor to the production of these goods. Each of the  $N$  households is paid a wage,  $w_i$ , for working at subcenter  $i$  and buys one unit of variant  $k$  at price,  $p_k$ . Both prices and wages will be determined by the model. Homogeneous labor is supplied at the center for a unit wage. The price of the homogeneous consumption good, the price of intermediate deliveries and the price of delivering the fixed private and public infrastructure are all equal to one. We will concentrate on the symmetric case where all subcenters are equidistant from the center, so that commuting and shopping travel times are identical and positive ( $t_k = t_i = t > 0$ ). Using the household budget equation to substitute for consumption of the homogeneous good, an indirect conditional utility function can

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<sup>3</sup> As  $N$  is large this means that every firm will be directed by its shareholders to maximise net profits.

be derived to express household preferences. This utility function is only defined in as far as one unit of the differentiated good is consumed and one unit of differentiated labor is supplied. In the absence of trip chaining, the following utility function represents the preferences of a household that buys differentiated good  $k$  and supplies labor to subcenter  $i$ :

$$U_{ik} = \underbrace{\tilde{h}_k - p_k - \delta\alpha^d t}_{A} + \underbrace{w_i - \tilde{\beta}_i - \alpha^w t}_{B} + \underbrace{\theta(1 - \beta) + \frac{1}{N} \sum_l \pi_l - T}_{C}. \quad (2)$$

The first three terms (A) represent the net utility from consuming differentiated good  $k$  with intrinsic quality component or willingness to pay  $\tilde{h}_k$ , that is bought at a price,  $p_k$ . and this requires a travel cost  $\alpha^d t$  to subcenter  $k$ . Note that for the consumer who trip chains, this travel cost is zero ( $\delta = 1$ ). The next three terms (B) are related to the supply of differentiated labor to subcenter  $i$ . This generates a wage  $w_i$  but has a disutility  $\tilde{\beta}_i$  and requires a travel cost  $\alpha^w t$ . The three last terms (C) have to do with the consumption of the homogenous good (before subtracting the transport costs). As the disutility of homogenous labor equals  $\beta$ , the first term in C represents the net utility derived from his supply of  $\theta$  units of homogenous labor for a unit wage. The second one represents the consumption possibilities derived from his equal share in total profits ( $\pi$ ) and the last term is the head tax. The net utility derived from the consumption of the homogenous good equals the terms in (C) plus  $w_i - \alpha^w t - p_k - \delta\alpha^d t$ , the net wage received from the supply of differentiated labor in  $i$  minus the costs of buying differentiated good  $k$ . Since the travel time required for shopping activities,  $t$ , is zero if this consumer trip chains, this translates into a higher consumption of the homogenous good.

We assume that all households value the quality of all product variants in the same way and experience the same disinclination to work at all subcenters. We set these valuations to zero without loss of generality. However, the households will still vary in their tastes. The utility of consumption of differentiated product variant  $k$  is then simply given by a stochastic component:  $\mu^d \varepsilon_k$ , such that

$$\tilde{h}_k = \mu^d \varepsilon_k \quad (3)$$

and the disutility of labor at subcenter  $i$  is similarly given by

$$\tilde{\beta}_i = -\mu^w \varepsilon_i. \quad (4)$$

The parameters  $\varepsilon_i$  and  $\varepsilon_k$  represent the intrinsic heterogeneity of household preferences and are assumed to be *i.i.d.* double exponentially distributed with mean normalised to zero and unit variance. The degree of heterogeneity of preferences is determined by  $\mu^w$  and  $\mu^d$ .

Substitution of (3) and (4) in the utility formulation (2), results in a random utility function for which the choice probabilities can be determined using the nested logit model.

### 3.3. *Nested logit model*

To derive the probabilities of working and shopping at a given subcenter we use an intuitive approach initially proposed by Ben-Akiva & Lerman (1979): the resident first selects his workplace with associated heterogeneity parameter  $\mu^w$  and then chooses where to shop with the associated heterogeneity parameter  $\mu^d$ . The decision tree for the nested logit is shown in Figure 5: at the top level, the individuals select their workplace while at the lower level they select their shopping location. The consumer surplus associated with the resident's shopping alternatives, given his work location, affects his initial workplace choice. As shown by Ben-Akiva & Lerman (1979), the nested logit approach requires for consistency that  $0 < \mu^d / \mu^w \leq 1$ , so that households' preferences for their choice of workplace are at least as strong as their preferences for shopping location.<sup>4</sup> The choice probabilities can also be obtained as the result of a maximization problem using the Generalized Extreme Value (GEV) approach of McFadden (1978).

**(Figure 5 approx here)**

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<sup>4</sup> In the extreme case where individuals do not care where they work ( $\mu^w \rightarrow 0$ ), everybody trip chains in our model.

When  $\frac{\mu^d}{\mu^w} > 1$  we could reformulate the full model and define a nested utility function where the shopping decision

comes first. All the results of this paper would carry over.

In order to simplify the exposition, we will concentrate on the price  $p_1$  and wage  $w_1$  set by firm 1 and assume that the prices and wages set by all other firms are identical and equal  $p^*$  and  $w^*$  respectively. We first develop the “shopping model” that determines the subutility a resident gets from shopping, given that he works in a given subcenter. This will be an input in the development of the “employment model” that selects the workplace. Once we have a full model for the household shopping and workplace decisions, we proceed to market clearing and firm behavior.

### 3.4. Shopping model

We first consider the shopping model. Here and in subsequent sections the following notation is used: superscripts  $s$  and  $w$  stand for shopping and working decisions; subscripts refer to decisions relating to subcenter 1 (denoted by 1) or other subcenters (denoted by -1); and subscripts for conditional decisions are further denoted 1/1 or 1/-1, representing shopping at subcenter 1, given that a resident works at subcenter 1 or elsewhere, respectively. The probability of a resident shopping at subcenter 1 that charges a price  $p_1$ , given he works there and given that all other subcenters charge a price  $p^*$ , can be expressed as

$$P_{1|1}^s(p_1, p^*) = \exp(-p_1/\mu^d) / \chi_{1|1}^s, \quad (5)$$

where  $\chi_{1|1}^s = \exp(-p_1/\mu^d) + (n-1)\exp((-p^* - \alpha^d t)/\mu^d)$ . The exponents of the terms in (5) represent the utility derived from shopping by the resident who works at subcenter 1. Thus, the first term of  $\chi_{1|1}^s$  (and the numerator of (5)) refers to a resident who trip chains, working and shopping at subcenter 1 (where price  $p_1$  is charged), while the second refers to the resident who works at subcenter 1 but shops elsewhere. The consumer surplus associated with the shopping decisions of a resident who works at subcenter 1, wherever he shops, can be calculated from the expected maximum utility derived from his shopping activities (see Anderson et al., 1992, Ben-Akiva and Lerman, 1985) and takes the form

$$CS_1^s = \mu^d \log \left[ \chi_{1|1}^s \right]. \quad (6)$$

The corresponding probability of a resident shopping at subcenter 1, given he does not work there is given by

$$P_{1|1}^s(p_1, p^*) = \exp \left( \left( -p_1 - \alpha^d t \right) / \mu^d \right) / \chi_{1|1}^s, \quad (7)$$

where  $\chi_{1|1}^s = \exp \left( \left( -p_1 - \alpha^d t \right) / \mu^d \right) + \exp \left( -p^* / \mu^d \right) + (n-2) \exp \left( \left( -p^* - \alpha^d t \right) / \mu^d \right)$ . Since all subcenters with the exception of subcenter 1 are identical, only one expression is needed. In this case the terms in  $\chi_{1|1}^s$  cover the options of: a) shopping at subcenter 1 but working elsewhere so there is a travel cost; b) combining shopping and working at some other subcenter ( $k \neq 1$  say); and c) shopping at  $k$  but working at subcenter  $j$  ( $j \neq k$  or 1), so there is a travel cost component. The resident has to travel to subcenter 1, so  $t$  appears in the numerator. Again the consumer surplus associated with shopping activities is calculated from the log of  $\chi_{1|1}^s$ .

The consumer surplus,  $CS_{-1}^s$ , for the shopping activity of a resident who works at any other subcenter  $k$ , ( $k \neq 1$ ) is

$$CS_{-1}^s = \mu^d \log \left[ \chi_{1|-1}^s \right]. \quad (8)$$

### 3.5. Employment model

The utility of an individual working at subcenter 1 is:

$$U_1^w = w_1 - \alpha^w t + CS_1^s + \mu^w \varepsilon_1,$$

where the parameter  $\varepsilon_1$  represents the intrinsic heterogeneity of household preferences for working at subcenter 1 (see Section 3.2). The probability of working at subcenter 1 is given by a nested logit model, as follows:

$$P_1^w(w_1, w^*) = \exp \left( \frac{w_1 - \alpha^w t + CS_1^s}{\mu^w} \right) / \chi_{1|1}^w, \quad (9)$$



where  $\chi_{1|1}^w = \exp\left(\left(w_1 - \alpha^w t + CS_1^s\right)/\mu^w\right) + (n-1)\exp\left(\left(w^* - \alpha^w t + CS_{-1}^s\right)/\mu^w\right)$  and  $CS_1^s$  is defined in (6) as the total consumer surplus from shopping of a household that works in subcenter 1. The probability of working at a subcenter other than subcenter 1 is given by

$$P_{-1}^w(w_1, w^*) = \exp\left(\frac{w^* - \alpha^w t + CS_{-1}^s}{\mu^w}\right) / \chi_{1|1}^w. \quad (10)$$

The denominator ( $\chi_{1|1}^w$ ) is the same as in (9) since the consumer still has the same chance of working at subcenter 1 and being paid  $w_1$  or working at another subcenter and being paid  $w^*$ .  $CS_{-1}^s$  is defined in (8) as the consumer surplus from shopping of a household who does not work at subcenter 1.

### 3.6. Market clearing

Let  $N_1^w$  be the proportion of households that work at subcenter 1 and  $N_1^s$  the proportion that shop there. We assume<sup>5</sup> that markets clear in every elementary period, and this implies, for each firm, that the number of workers equals total sales<sup>6</sup> so that

$$N_1^w = N_1^s. \quad (11)$$

This assumption means that prices and wages are perfectly flexible (so that there is no inventory, unemployment or shortages). We can further express the number of shoppers frequenting subcenter 1 as

$$N_1^s = N_1^w P_{1|1}^s + N(1 - P_1^w) P_{1|-1}^s, \quad (12)$$

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<sup>5</sup> One could also replace the assumption by an explicit derivation of the equilibrium for, say, perishable goods or alternatively introduce storage etc.. Firms need to hire labor to produce, and then the quantity sold is the minimum of the quantity demanded and produced. The firm maximizes profits by hiring exactly enough labor for the expected sales and the equilibrium with flexible prices and wages involves a direct relation between number of workers and shoppers that will later translate into a relation between the price and wage of every firm.

where  $P_1^w$  is the probability of working at subcenter 1, and  $P_{1|1}^s$  and  $P_{1|-1}^s$  respectively denote the probability of a resident shopping at subcenter 1, given that he does or does not work there.

Then, since by definition  $N_1^w = NP_1^w$ , (12) simplifies to

$$P_1^w \left[ 1 - P_{1|1}^s + P_{1|-1}^s \right] - P_{1|-1}^s = 0, \quad (13)$$

which provides an implicit relation between the price  $p_1$  and wage  $w_1$  set by firm 1. This relation means that, in an equilibrium, a firm that wants to cut its price and gain market share will need to increase its wage in order to produce the extra goods it wants to sell. Equation (13) is still in implicit form; its implications for the behavior of the firm are explained in the following sections.

### 3.7. Firms' Behavior

In general, the profit of firm  $i$  can be written

$$\pi_i(w, p) = (p_i - w_i - c^1 - \alpha^h t) NP_i^w - (F + S) \quad \forall i = 1 \dots n, \quad (14)$$

where the demand  $D_i = NP_i^d = NP_i^w$  under market clearing conditions<sup>7</sup>.

Firms compete in a non-cooperative Nash game with their own prices and wages as the strategic variables. Since from (13) we know that  $p_1$  determines  $w_1$  and vice versa, we take the wage as the strategic variable for firm 1 and write  $p_1 = g_1(w_1)$ . Note that all firms other than firm 1 charge  $p^*$  for their product and pay wage  $w^*$ . Then, further assuming that firm 1 takes the prices and wages of the other firms as given, the first order condition for profit maximization by this firm is given by

$$\frac{d\pi_1}{dw_1} = \left[ \left( \frac{dg_1}{dw_1} - 1 \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( \frac{1 - P_1^w}{\mu^w} \right) \right] NP_1^w = 0. \quad (15)$$

---

<sup>6</sup> Remember the assumption that the production of one unit of the differentiated good requires one unit of differentiated labor.

<sup>7</sup>  $P_i^w$  and  $P_i^d$  are the probability of working and shopping at any subcenter  $i$ , respectively.

In Section 3.9, we derive an expression for the key strategic term  $dg_1/dw_1$  in the case with trip chaining. As we want later to compare the equilibria with and without trip chaining, in the next section, we recall some properties of the non trip chaining equilibrium derived in de Palma and Proost (2006).

### 3.8. Market equilibrium without trip chaining

Recall Propositions 1 and 3 in de Palma and Proost (2006) for the no congestion case.

**Proposition (price)** *When no trip chaining is permitted, there exists a unique symmetric short run Nash equilibrium in prices and wages. The price-wage equilibrium is given by*

$$p_{ntc}^* - w_{ntc}^* = c^1 + \alpha^h t + (\mu^w + \mu^d) \frac{n}{n-1}. \quad (16)$$

In the next section (see discussion after Proposition 1) we will show that this proposition is a limiting case of the equilibrium with trip chaining.

**Proposition (free entry)** *In the free entry symmetric Nash equilibrium with no trip chaining permitted and when firms pay a levy equal to the public infrastructure cost ( $S=K$ ) there is one subcenter too many.*

In this equilibrium, trip-chaining does not occur and households make separate working and shopping trips, although these may be to the same destination. We denote this as reference equilibrium and in later sections we compare the results for the reference equilibrium with the results for the trip-chaining equilibrium.

### 3.9. Short run market equilibrium with trip chaining

We can now make a full analysis of the equilibrium in the case trip chaining is possible. In order to show the properties of this equilibrium we start with the symmetric price equilibrium with trip-chaining in which all firms charge  $p^*$  and pay  $w^*$ . We consider single price and wage deviations from this equilibrium. We first suppose that firm 1 deviates and sets price  $p_1$  for its product and pays its workers a wage  $w_1$ . All other firms continue to charge  $p^*$  and pay  $w^*$ .

Market clearing in fact precludes any other possible deviations since, as we have seen, a change in wage by one firm must be accompanied by a price alteration at the same firm. Analysis of the behavior of firm 1 allows us to derive the conditions for the symmetric price equilibrium with trip-chaining.

Since firm 1 deviates, his profit becomes

$$\pi_1(w_1, w^*, p_1, p^*) = (p_1 - w_1 - c^1 - \alpha^h t) N P_1^w - (F + S). \quad (17)$$

In order to derive an expression for a candidate Nash equilibrium from the profit maximization condition and prove its existence, we first need to determine the derivative of the price at firm 1 with respect to its wage ( $dg_1/dw_1$  in (15)). Defining  $\mu \equiv \mu^d / \mu^w \leq 1$  and  $\Phi(p_1, p^*) \equiv P_{1|1}^s - P_{1|-1}^s > 0$ , we have

**Lemma 1**  $\frac{dg_1}{dw_1} = \frac{-\mu}{1 + (1 - \mu)\Phi} < 0.$

Proof. See Appendix A1.

To understand Lemma 1, first consider the case without trip chaining. Then  $dg_1/dw_1|_{nc} = -\mu$  because  $\Phi=0$ , which means that the more loyal consumers are to their variety of product, the larger the price cut needed to sell the extra production brought about by the workers attracted by a wage increase. When trip chaining is permitted, the necessary price cut is smaller because the extra workers attracted by a wage increase will actually trip chain themselves, so fewer new customers need to be attracted. There is a greater probability of trip chaining than of working and shopping in separate locations.

Substitution of  $dg_1/dw_1$  from Lemma 1 in (15) and replacing  $P_1^w$  in terms of the conditional shopping probabilities ( $P_{1|1}^s$  and  $P_{1|-1}^s$ ) from (13), we obtain

$$\left[ \left( \frac{-(1 + \mu) - (1 - \mu)\Phi}{1 + (1 - \mu)\Phi} \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( \frac{1 - P_{1|1}^s}{\mu^w [1 - \Phi]} \right) \right] N \frac{P_{1|-1}^s}{[1 - \Phi]} = 0. \quad (18)$$

Now, at equilibrium in the symmetric case,  $p_1 = p^*$  and we can therefore rewrite the conditional shopping probabilities (5) and (7) (where (5) represents  $\delta$ , the share of shoppers that are trip chaining) as

$$P_{1|1}^s = \frac{1}{1 + (n-1)\lambda} > \frac{1}{n}, \quad (19)$$

and

$$P_{1|-1}^s = \frac{\lambda}{1 + (n-1)\lambda} < \frac{1}{n}, \quad (20)$$

where  $\lambda \equiv e^{-\alpha^d t / \mu^d} \in (0, 1)$  from our model assumptions.  $\lambda$  can be seen as a trip chaining cost parameter. When travel costs (frequency of shopping  $\alpha^d$  times unit transport cost  $t$ ) are high and shopping and variety preferences are not strong,  $\lambda$  is small and there is more frequent trip chaining ( $\delta$  is high). When, on the other hand, travel costs are low but love for variety is high, one may see much more trip chaining as the cost of extra trips is small compared to the gain in variety. Note that total transport costs for shopping amount to  $(1 - P_{1|1}^s) \alpha^d t N$  (see (1) where  $\delta = P_{1|1}^s$  and  $N=D$ ). Thus, we can write

$$\Phi = \frac{1 - \lambda}{1 + (n-1)\lambda}. \quad (21)$$

Substitution of expressions (19), (20) and (21) in (18) allow us to specify the candidate Nash equilibrium.

**Proposition 1** *When trip chaining is permitted, there exists a unique symmetric short run Nash equilibrium in prices and wages. The price-wage equilibrium is given by*

$$p^* - w^* = c^1 + \alpha^h t + (\mu^w + \mu^d) \frac{n}{n-1} - \frac{n\mu^d}{n-1} \left\{ \frac{(1-\mu)(1-\lambda)}{[2-\mu + (n-2)\lambda + \mu\lambda]} \right\}. \quad (22)$$

Proof. See Appendix A2.

In Appendix 2 we show that when  $n-1$  firms are charging the candidate price equilibrium, the last firms' profit equilibrium is globally quasi-concave. This means that no firm has an incentive

to suddenly jump to a higher price in order to focus on consumers in the same subcenter. This proves that there is no asymmetric equilibrium.

From (14), in equilibrium, a firm's profit per household is

$$\pi^* = \frac{(\mu^w + \mu^d)}{n-1} - \frac{\mu^d}{n-1} \left[ \frac{(1-\mu)(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda)} \right] - \frac{F+S}{N}. \quad (23)$$

Using the fact that  $\mu < 1$  and  $\lambda < 1$ , it can be verified that the gross profit (neglecting fixed costs) is positive. The comparative statics result is straightforward and left to the reader. The relationship between the mark-up (price minus wage), profit and the parameters  $n, \alpha^d, \mu^w, \mu^d$  and  $t$  is discussed in Section 5 using a numerical example.

It is possible to perform the same analysis, within the nested logit framework, for the case where consumers have to perform two single purpose trips, even if they work and shop at the same subcenter. This is the reference equilibrium without trip chaining, which is the same as the equilibrium which can be derived when working and shopping decisions are taken independently (see (16)), with the restriction  $\mu^w / \mu^d > 1$  for the nested logit approach (Anderson et al., 1992). In this case profits only depend on the household heterogeneity parameters and number of firms. We can now compare the symmetric short run trip chaining equilibrium, (22), with the symmetric short run, reference equilibrium, (16).

**Proposition 2** *The symmetric short run firm mark-up when households can trip chain cannot exceed the mark-up when households can only perform single purpose trips. The mark-ups are equal when  $\mu^d = \mu^w$ . The difference in mark-up is given by*

$$(p^* - w^*) - (p_{ntc}^* - w_{ntc}^*) = -\frac{n\mu^d(1-\mu)}{n-1} \left[ \frac{(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda)} \right] < 0. \quad (24)$$

The intuition as to why trip chaining decreases margins is not obvious given the complexity of the RHS of (24). The dominant mechanism can be seen as follows. Compared to the no trip chaining case, the same price decrease will attract more customers because a relatively large part ( $> 1/n$ ) of the necessary labor to produce it trip chains and adds to the group of customers.

This means there is a larger reward for a price cut (a flatter demand curve) and this will lead to more price cuts and ultimately lower profit margins.

Compared to previous discussions of the effect of trip chaining, as in Raith (1996), we see that the explicit integration of the labor market in a general equilibrium model adds an extra effect: price decreases that generate extra sales need workers to produce them and as these are potential customers this affects price setting behavior.

### 3.10. The long run free entry equilibrium with trip chaining

Using (23) we can write the difference in profit per household as

$$\pi^* - \pi_{ntc}^* = -\frac{\mu^d}{n-1} \left[ \frac{(1-\mu)(1-\lambda)}{(2-\mu+(n-2)\lambda+\mu\lambda)} \right] < 0. \quad (25)$$

It follows directly from (25) that, for any given number of firms  $n$ , the profit level of firms present in the market when trip chaining is possible always lies below the corresponding level when trip chaining is not possible. At free entry, the profit of all firms in the market is zero. Hence this must occur at a smaller value of  $n$  when there is trip chaining.

**Proposition 3** *The symmetric long run Nash equilibrium with free entry has a smaller number of firms when trip chaining is possible than when trip chaining is not possible:  $n^f < n_{ntc}^f$ .*

We will illustrate this difference in the entry of firms numerically in Section 5.

## 4. WELFARE ANALYSIS

### 4.1. Consumer Surplus in the short run

Using the expected maximum utility approach described in Sections 3.4 and 3.5, the total consumer surplus, from working and shopping activities associated with households working at subcenter 1, can be obtained from the log of the denominator of (9). In the symmetric equilibrium, this is indeed the consumer surplus of households working at any subcenter and can be written as

$$CS = \mu^w \log \left[ n \exp \left( \frac{w^* - \alpha^w t + CS^s}{\mu^w} \right) \right], \quad (26)$$

where  $CS^s = -p^* + \mu^d \log[1 + (n-1)\lambda]$  is the consumer surplus<sup>8</sup> derived by households from shopping activities ((6) and (8)) in the symmetric equilibrium with trip chaining. An equivalent expression can be obtained for the reference case without trip chaining (see Appendix A3).

**Proposition 4** *In the symmetric short run Nash equilibrium, the consumer surplus when households can trip chain is larger than the consumer surplus when households must perform only single purpose trips. The difference in consumer surplus is given by*

$$CS - CS_{ntc} = (p_{ntc}^* - w_{ntc}^*) - (p^* - w^*) + \mu^d \log \left[ 1 + \frac{\lambda^{-1} - 1}{n} \right] > 0. \quad (27)$$

Proof. See Appendix A3.

A higher mark up ( $p-w$ ) for the firm means a higher price and lower wage for the differentiated good and thus a lower consumer surplus for the household. The second travel cost component can be clarified by further subdividing this term into the travel time saving for households who do not change their behavior between the two equilibria and the cost for a household that changes its shopping behavior to take advantage of trip chaining. Hence

$$\mu^d \log \left[ 1 + \frac{\lambda^{-1} - 1}{n} \right] = \frac{\alpha^d t}{n} + \mu^d \log \left[ \lambda^{\frac{1}{n}} \left( 1 + \frac{\lambda^{-1} - 1}{n} \right) \right] \quad (28)$$

The first term on the RHS of (28) is equal to the transportation cost saving when an individual works in a subcenter and has his most preferred good in the same subcenter (this event happens with probability  $1/n$ ). The second term is the saving from economizing on transport costs when the first best choice is not at the same location as the work place but close enough in the idiosyncratic preference space (see Anderson et al., 1989). This second term, which translates the quality adjustment of the consumer, is strictly positive and converges to zero when the travel time goes to zero and  $\lambda$  goes to one.

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<sup>8</sup> Consumer surplus and welfare are calculated per household.



Substitution of the mark-up terms from (24) into (27) allows us to perform a comparative statics exercise on the model parameters. It can easily be shown that the difference in consumer surplus is decreasing in the trip chaining cost  $\lambda$  and  $n$ . These results are verified by the numerical exercise in Section 5.

## 4.2. Welfare effects in the short run

For a quasi linear utility function, welfare difference is the sum of the difference in consumer surplus from the consumption and supply of differentiated product and differentiated labor plus the difference in producer surplus from the supply of the differentiated product. Using expression (27) and the fact that profits are redistributed equally to households, we obtain:

**Proposition 5** *In the symmetric short run Nash equilibrium, welfare<sup>8</sup> is greater when households can trip chain, than when they have to perform only single purpose trips. The difference in welfare is given by*

$$W - W_{ntc} = \mu^d \log \left[ 1 + \frac{\lambda^{-1} - 1}{n} \right]. \quad (29)$$

Clearly the difference in welfare between the equilibria with and without trip chaining is equal to the difference in consumer surplus minus the difference in profits. Equation (29) is positive since with trip chaining the individuals have more options (to use or not use the trip chaining scheme). Therefore (29) represents, in a sense, the option value associated with trip chaining. Note that the fact that prices are adjusted by trip chaining is irrelevant for the welfare analysis, since price changes are pure transfers with no social impacts, at least in our model with fixed total demand and supply for the differentiated good and differentiated labor.

With price elastic demand we conjecture that the welfare effects of allowing trip chaining are expected to be larger as trip chaining reduces average transport costs and increases consumption of differentiated products.

### 4.3. Welfare effects in the long run

When we analyze the welfare effects of trip chaining in the long run, there are three conflicting forces to consider. For an identical number of firms (short run) we know that the gain in transport costs is larger than the loss in consumer diversity (Proposition 5). But, we also know from Proposition 3 that the free entry number of firms is smaller with trip chaining than without trip chaining and this means a loss of diversity (welfare loss) and a decrease in fixed costs (welfare gain).

We will be able to show that trip chaining is indeed welfare enhancing even with free entry but this will require a few intermediate steps contained in Lemmas 2 and 3. These are proved in Appendices A4 and A5.

**Lemma 2** The welfare maximizing number of firms is smaller when the trip chaining option is possible:  $n^0 < n_{ntc}^0$ .

The next intermediate result we need is to show that, with the trip chaining option, the free entry equilibrium always has too many firms compared to the optimal number of firms.

**Lemma 3** When trip chaining is possible (and given a tax per firm equal to  $K$ ), the free entry number of firms is larger than the optimal number of firms:  $n^f > n^0$ .

We can now prove the following proposition, which extends Proposition 5 to the long run.

**Proposition 6** *In the free entry Nash equilibrium, the welfare is higher when trip chaining is possible than when it is not.*

$$W(n^0) \geq W(n^f) \geq W_{ntc}(n_{ntc}^0) \geq W_{ntc}(n_{ntc}^f). \quad (30)$$

#### Proof

Recall from Section 3.10 that the profit curve for firms when trip chaining is possible always lies below that when trip chaining is not possible, as shown on Figure 6. Further, from Proposition 3 the free entry number of firms with trip chaining is smaller than without trip chaining ( $n^f < n_{ntc}^f$ ). Using Lemma 2 and Lemma 3, we also have  $n^0 \leq n^f \leq n_{ntc}^f$ . We know from (29) (Proposition 5) that the welfare curve when there is trip chaining always lies above that when

trip chaining is not possible. These curves are also illustrated in Figure 6. We can then show that (30) holds. The first inequality in (30) follows from the definition of an optimum, the next inequality follows from the concavity of  $W$ , the last inequality follows again from the definition of an optimum in the case of no trip chaining. QED.

**(Figure 6 approx here)**

## 5. NUMERICAL EXAMPLE

The trip chaining equilibrium in price and wages, (24), depends in a complex way on a number of parameters: in particular, the preferences for variety of work place ( $\mu^w$ ), the preference for variety in the product market ( $\mu^d$ ), the frequency of shopping trips ( $\alpha^d$ ), travel time,  $t$  and the number of firms  $n$ . The following numerical exercise illustrates how these parameters influence the effect of the trip chaining option on the number of households trip chaining, the price-wage equilibrium, profit, consumer surplus and welfare, and gives orders of magnitude of the different effects.

We use the simple, stylized example of an economy of one day. We have a metropolitan area of one million inhabitants. As a reference, for the short run equilibrium, we assume there are three firms or subcenters offering the differentiated good. Each resident makes one commuting trip and one shopping trip per day, each of the two trips takes in total half an hour every day. He/she also supplies 7.5 hours of labor, of which one hour is spent on the production of the differentiated good. Production of the intermediate good requires 0.1 units of homogeneous labor. Finally, we set the total fixed costs<sup>9</sup> per firm at 0.5 hours of labor per capita or some 30% of average costs. These data mean that the value added in the differentiated good industry represents roughly 20% of GDP.

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<sup>9</sup> Fixed costs here include levies (S) which are assumed equal to the fixed public inputs (K) since there are no head taxes (T).

In Table 1 we examine, for the short run equilibrium with a fixed number of firms, the effect on price minus wage and on gross profit ( $\pi$ ) of varying exogenous factors like the consumers' preference for work and shopping locations ( $\mu^w$  and  $\mu^d$ , respectively), number of shopping trips ( $\alpha^d$ ) and travel time. We do this for the equilibrium with and without trip chaining. In the last line, we also look at the effect of increasing the number of firms. The short run equilibria (with given number of firms) are presented in the middle part of Table 1. The long run equilibrium with "free entry" number of firms are given in the last two columns.

We first examine the short run equilibria. Consider the first line of Table 1, where we see the effect of the reference values on the number of trip chainers (45%), on the profit margins ( $p-w$ ) that decrease and on profits that decrease by 2.7%. This is our reference for a comparative statics analysis of the effect of certain parameters on the trip chaining. On the second line of Table 1, we see that a stronger preference for the workplace location ( $\mu^w = 5 > 2$ ) increases overall profits because firms can more easily lower wages before losing workers. The number of trip chainers (45%), however, is not affected, as trip chaining basically means giving up product preference in exchange for lower transport costs. On the third line we test the effect of weaker preferences for the product location (smaller  $\mu^d = 0.1 < 1$ ). As consumers now care much less about the product variant they buy, firms are forced to charge lower prices to retain shoppers. The overall level of profits is decreased by some 33% (remember that the profit level in these models with differentiated product and labor market is proportional to  $\mu^d + \mu^w$  [cfr.(16) and (22)] and this goes down from value 1+2 to value 2.1). In this case, all consumers shop at the workplace (99% trip chaining) as they have very low preference for product variety. The effect on profits of allowing trip chaining remains of the same order (2.3%). In the fourth line of Table 1 we drop the frequency of shopping by a factor of five ( $\alpha^d = 0.2 < 1$ ). This reduces the transport benefits from trip chaining: instead of saving five trips per week, one saves one trip per week. The number of households trip chaining decreases from 45% to 36% and profits hardly change because the competition inducing effect of the trip chaining option is no longer there. The opposite happens on line 5, where we have multiplied the travel time by a factor of four. Now the transport cost savings of trip chaining become very important and 79% trip chain if they are

allowed to. Increasing or decreasing the transport costs does not affect the overall profit level when trip chaining is not allowed because overall demand is fixed and transport costs are identical for all firms. But, when trip chaining is allowed, the profits of firms are more vulnerable to trip chaining when transport costs are high. The intuition is that a price decrease generates extra demand but this extra demand also requires workers. When a large fraction of the extra workers are trip chaining (because of high transport costs), the same price decrease attracts many more consumers, competition is fiercer and profit margins decrease. In the last line of Table 1 we increase the existing number of firms ( $n$ ) from 3 to 10. Profits per firm decrease because there are more competitors. Now 15% of the population combines the work and shopping trip. This can be compared to the 10% ( $1/n$ ) who would shop and work and the same subcenter, making two separate trips, when trip chaining is not allowed.

Exogenous parameters					Short run equilibria						Free entry	
$\mu^d$	$\mu^w$	$\alpha^d$	$t$	$n$	% trip chaining	$p^*-w^*$	$p_{ntc}^*-w_{ntc}^*$	$\pi^*$	$\pi_{ntc}$	$\Delta\pi^*$ (%GDP)	$n^f$	$n_{ntc}^f$
1	2	1	0,5	3	45%	4,488	4,610	1,459	1,500	0.48	6	7
1	5	1	0,5	3	45%	8,923	9,110	2,938	3,000	0.73	12	13
0,1	2	1	0,5	3	99%	3,188	3,260	1,026	1,050	0.28	5	5
1	2	0,2	0,5	3	36%	4,585	4,610	1,492	1,500	0.10	6	7
1	2	1	2	3	79%	4,259	4,640	1,373	1,500	1.49	6	7
1	2	1	0,5	10	15%	3,410	3,443	0,330	0,333	0.04		

**Table 1<sup>10</sup> Comparative statics with and without trip chaining in the short run and in the long run**

<sup>10</sup> For the short run equilibria we present in table 1:  $p^*-w^*$ , the mark-up when there is trip chaining (see eqn 22);  $p_{ntc}^*-w_{ntc}^*$ , the mark-up when no trip chaining (see eqn 16);  $\pi^*$ , profit per household when trip chaining, (eqn 14 or 23 with fixed costs neglected);  $\pi_{ntc}$ , profit per household for no

It is clear from Table 1 that when consumers can trip chain, firms cannot make greater profits than when consumers can only make single purpose trips. The magnitude of the difference in profits obviously depends on the values of the input parameters but the difference is large for long travel times or high frequency of shopping trips. With free entry (last two columns of Table 1), we see that trip chaining reduces the number of firms. The effect of travel time and trip frequency is much smaller than that of consumer heterogeneity and is not apparent when integer numbers of firms are considered, as is the case here.

Table 1 has taught us that firms do not in general like the possibility of trip chaining. They may like it individually in order to gain a competitive advantage over their rivals but if everyone does it, profits decrease. In order to know whether, for society as a whole, trip chaining is beneficial, we need to examine the total welfare effects and compare the gains with the cost of extra investments in facilities (parking, public transit etc.) and work practices that may be necessary. In Table 2 we present the difference in consumer surplus and welfare (per household) between the two equilibria in the short and long run. All entries are shown as a share of GDP. Remember that welfare is the sum of consumer surplus plus producer surplus (that pays for the fixed costs) minus the cost of extra facilities for trip chaining. The latter term is set to zero, so our welfare outcome is a gross concept. In the first line of Table 2 (reference case), we see that the gross welfare gain of trip chaining (0.20) is smaller than the gain in transport costs ( $0.23 = \delta \alpha^d t$ ) and smaller than the gain in consumer surplus (that includes price decreases and travel cost savings but also accounts for giving up preferred product variety when trip chaining). As shown in propositions 5 and 6, (gross) welfare is always higher when trip chaining is allowed than when it is not and this holds in the short run and in the long run. In our simulations the gross welfare gain in the short run accounts for between 75 and 90% of the gain in transport costs.

The short and long run gross gains from trip chaining are of the order of 0.4 to 4.6% of GDP to be compared with the share of 20% of the differentiated good in GDP and with the one hour of transport time per day for shopping and commuting. These gains are clearly non negligible but

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trip chaining (also directly from (14) or (25) with no fixed costs); and  $\Delta\pi$ , the difference in profit calculated as a percentage of GDP.

they are gross and need to be compared with the extra investments that may be necessary to make trip chaining possible.

Exogenous parameters				Short run equilibria				Free entry
$\mu^d$	$\mu^w$	$\alpha^d$	$t$ (hrs)	$n$	$\delta\alpha^d t$ (%GDP)	$CS - CS_{ntc}$ (%GDP)	$W - W_{ntc}$ (%GDP)	$W(n^f) - W_{ntc}(n_{ntc}^f)$ (%GDP)
1	2	1	0,5	3	2.66	3.74	2,3	1,65
1	5	1	0,5	3	2.66	4.50	2,3	0,9
0,1	2	1	0,5	3	5.80	5.46	4,6	4,0
1	2	0,2	0,5	3	0.42	0.70	0,4	0,6
1	2	1	2	3	18.52	17.90	13,4	9,0
1	2	1	0,5	10	0.91	1.13	0,7	1,65

**Table 2 Welfare effects with and without trip chaining**

## 6. CONCLUSIONS

In this paper we studied the effect of trip chaining on the profits and number of firms as well as on welfare. We found that firms make smaller profits when the trip chaining is permitted because price cuts tend to generate higher demand responses. The higher demand responses come from the workers that are attracted to produce the extra demand and some of them also become consumers of the firm. In the symmetric equilibrium, lower profit margins imply that a smaller number of firms can survive in the free entry case. Welfare does unambiguously increase in the short run and with free entry because the gain in transport costs is not fully offset by the loss of variety of firms.

Trip chaining is beneficial to consumers in the short and long run. On the contrary, firms collectively lose when trip chaining is possible and would therefore not support legislation promoting it. However, the net impact (i.e. the impact on welfare) is positive, which obviously suggests such legislation should be encouraged (via parking policies, pricing and information

systems, for example) if its costs are limited. In the numerical example we constructed, the differentiated goods sector represented some 20% of GDP and the gross gain of allowing trip chaining varied between 0.5 and 4% of GDP.

In this paper we compared a symmetric equilibrium where trip chaining is allowed with a symmetric equilibrium where it is not. We can add one step to the game and consider trip chaining as an option to be decided unilaterally by each firm. Consider a situation where  $n-m$  firms allow trip chaining and the remaining  $m$  do not allow trip chaining. We look at the incentive for one of the  $m$  firms to change its policy and allow trip chaining. The profits of a firm that decides to allow trip chaining change for two reasons. First, keeping its price fixed, its demand increases as the transport cost reduction increases its attractiveness. Second, there will be new price equilibriums since the competition will decrease their price and since goods are strategic substitutes, the deviant firm will also decrease its price. We conjecture the latter effect is dominated by the former and as a result the profit of the deviant firm increases while the profit of the other  $n-1$  firms decreases<sup>11</sup>. We are typically facing a prisoners' dilemma situation (Thisse and Vives, 1998) and as a result all firms will accept trip chaining one after another and, as we have seen from Proposition 2, all firms will be worse off compared to the situation when trip chaining is not allowed.

For simplicity we have ruled out transport congestion. With congestion, trip chaining has an extra benefit for consumers since it reduces travel time for all consumers, not only those taking advantage of trip chaining. It is still the case that the firm adopting trip chaining gains a competitive advantage in the sense that the cost reduction for its consumers is larger than for consumers who patronize a firm which does not allow for trip chaining. As a consequence the unraveling result discussed above still holds. Moreover, the total welfare gain when all firms adopt trip chaining will still be larger than without it.

In the end, one may therefore expect that industry associations will lobby against trip chaining, for example, by relocating far enough from the business district to make trip chaining unfeasible. This is the same phenomenon as the opening hours discussion where each firm has an

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<sup>11</sup> Anderson and de Palma (2001) have shown that in an asymmetric logit duopoly, the high quality firm is also the higher profit one.



incentive to deviate and steal markets from its competitors by staying open longer (Rouwendal and Rietveld, 1999). However, at least when demand is inelastic, all firms will be worse off with extended opening hours.

Our results have been derived using a stylized city lay-out that offered maximum symmetry for the discussion of the price competition and the analysis of entry. Other city lay-outs tend to introduce a systematic competitive advantage for every firm in the competition for households located nearby. We conjecture that the flavor of our results carries over to these asymmetric settings but the analytical derivations need still to be done for each of the generic lay-outs one can think of.

The next research priority is to integrate the empirical research on the consumers' behavior side with our economic model and to estimate a full structural model of pricing, wage setting and trip chaining behavior.

## APPENDICES

### Appendix A1: Proof of Lemma 1.

Recall from (13) that

$$P_1^w \left[ 1 - P_{1|1}^s + P_{1|1-1}^s \right] - P_{1|1-1}^s = 0. \quad (31)$$

We will denote the left hand side of (31) by  $X$  so that  $X = 0$ . This expression is constant and can be differentiated implicitly to give

$$\frac{dg_1(w_1)}{dw_1} = - \frac{\partial X / \partial w_1}{\partial X / \partial p_1}. \quad (32)$$

We next evaluate the numerator and denominator of the right hand side of (32) by differentiating (31).

$$A) \quad \frac{\partial X}{\partial w_1} = \frac{\partial P_1^w}{\partial w_1} [1 - \Phi] \quad (\text{Recall } \Phi \equiv P_{1|1}^s - P_{1|1-1}^s.)$$

We can substitute  $\frac{\partial P_1^w}{\partial w_1} = \frac{1}{\mu^w} [1 - P_1^w]$  to obtain

$$\frac{\partial X}{\partial w_1} = \frac{1}{\mu^w} [1 - P_1^w] P_1^w [1 - \Phi]. \quad (33)$$

$$B) \quad \frac{\partial X}{\partial p_1} = \frac{\partial P_1^w}{\partial p_1} [1 - \Phi] - P_1^w \frac{\partial P_{1|1}^s}{\partial p_1} - (1 - P_1^w) \frac{\partial P_{1|1-1}^s}{\partial p_1}. \quad (34)$$

To evaluate this expression, we consider each of the partial derivatives on the right hand side in turn. Firstly

$$\frac{\partial P_1^w}{\partial p_1} = \frac{1}{\mu^w} \frac{\partial CS_1}{\partial p_1} [1 - P_1^w] P_1^w - \frac{1}{\mu^w} \frac{\partial CS_{-1}}{\partial p_1} (n-1) P_{-1}^w P_1^w. \quad (35)$$

The derivatives of the consumer surplus with respect to price are given by

$$\frac{\partial CS_1}{\partial p_1} = \mu^d \exp(-p_1/\mu^d) D_{1|1}^{s-1} \left( \frac{-1}{\mu^d} \right) = -P_{1|1}^s \text{ and}$$

$$\frac{\partial CS_{-1}}{\partial p_1} = \mu^d \exp((-p_1 - \alpha^d t)/\mu^d) D_{1|-1}^{s-1} \left( \frac{-1}{\mu^d} \right) = -P_{1|-1}^s \text{ with}$$

$$D_{1|1}^s = \exp(-p_1/\mu^d) + (n-1) \exp((-p^* - \alpha^d t)/\mu^d) \text{ and}$$

$$D_{1|-1}^s = \exp((-p_1 - \alpha^d t)/\mu^d) + \exp(-p^*/\mu^d) [1 + (n-2) \exp(-\alpha^d t/\mu^d)]. \text{ These are the}$$

expressions for Roy's identity in the case of a discrete choice model. Substituting these derivatives in (35) leads to

$$\frac{\partial P_1^w}{\partial p_1} = -\frac{[1 - P_1^w] P_1^w}{\mu^w} \Phi, \quad (36)$$

where we have also used  $(1 - P_1^w) = (n-1)P_{-1}^w$ .

Next we have

$$\frac{\partial P_{1|1}^s}{\partial p_1} = -\frac{1}{\mu^d} [1 - P_{1|1}^s] P_{1|1}^s \quad (37)$$

and

$$\frac{\partial P_{1|-1}^s}{\partial p_1} = -\frac{1}{\mu^d} [1 - P_{1|-1}^s] P_{1|-1}^s. \quad (38)$$

Substituting (36), (37) and (38) in (35), we obtain

$$\frac{\partial X}{\partial p_1} = -\frac{[1 - P_1^w] P_1^w}{\mu^w} \Phi [1 - \Phi] + \frac{P_1^w}{\mu^d} [1 - P_{1|1}^s] P_{1|1}^s + \frac{(1 - P_1^w)}{\mu^d} [1 - P_{1|-1}^s] P_{1|-1}^s. \quad (39)$$

Finally, substituting (33) and (39) in (32) leads to

$$\frac{dg_1}{dp_1} = -\frac{\frac{1}{\mu^w} [1 - P_1^w] P_1^w [1 - \Phi]}{-\frac{[1 - P_1^w] P_1^w}{\mu^w} \Phi [1 - \Phi] + \frac{P_1^w}{\mu^d} [1 - P_{1|1}^s] P_{1|1}^s + \frac{(1 - P_1^w)}{\mu^d} [1 - P_{1|-1}^s] P_{1|-1}^s}. \quad (40)$$

We can now use our original expression (31) to eliminate  $P_1^w$  from (40). Then, dividing numerator and denominator by  $[1 - \Phi]$  to simplify the equation, we obtain

$$\frac{dg_1}{dw_1} = \frac{-\mu}{1 + (1 - \mu)\Phi} \quad (41)$$

where  $\mu \equiv \frac{\mu^d}{\mu^w}$ . QED.

### Appendix A2: Proof of Proposition 1.

Recall from Lemma 1 that at the candidate equilibrium

$$\frac{dp_1}{dw_1} = \frac{-\mu}{1 + (1 - \mu)\Phi}, \quad (42)$$

where  $\mu \equiv \frac{\mu^d}{\mu^w}$  and  $\Phi \equiv P_{1||}^s - P_{1|-1}^s$ . This expression is negative and single valued, so that

there exists a one-to-one relationship between  $p_1$  and  $w_1$ . Hence the set of prices is a convex, compact set and the equilibrium exists. Further (42) is constant, since  $\mu$ ,  $t$  and  $n$  are all exogenous.

Since a candidate equilibrium exists, we need only show that the profit function is quasi-concave to guarantee that the candidate equilibrium is the unique Nash solution.

At any extremum

$$\frac{d\pi_1}{dw_1} = \left[ \left( \frac{dp_1}{dw_1} - 1 \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( \frac{1 - P_1^w}{\mu^w} \right) \right] NP_1^w = 0. \quad (43)$$

The corresponding second order condition is given by

$$\begin{aligned}
\frac{d^2 \pi_1}{dw_1^2} &= NP_1^w \frac{d^2 p_1}{dw_1^2} + NP_1^w \left[ \left( \frac{dp_1}{dw_1} - 1 \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( -\frac{P_1^w}{\mu^w} \right) \left( \frac{1 - P_1^w}{\mu^w} \right) \right] + \\
&\quad \left[ \left( \frac{dp_1}{dw_1} - 1 \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( \frac{1 - P_1^w}{\mu^w} \right) \right] NP_1^w \left( \frac{1 - P_1^w}{\mu^w} \right) \\
&= NP_1^w \frac{d^2 p_1}{dw_1^2} + NP_1^w \left( \frac{1 - P_1^w}{\mu^w} \right) \left[ 2 \left( \frac{dp_1}{dw_1} - 1 \right) + (p_1 - w_1 - c^1 - \alpha^h t) \left( \frac{1 - 2P_1^w}{\mu^w} \right) \right].
\end{aligned} \tag{44}$$

From (43) we can replace  $(p_1 - w_1 - c^1 - \alpha^h t)$  in (44) to get

$$\begin{aligned}
\frac{d^2 \pi_1}{dw_1^2} &= NP_1^w \frac{d^2 p_1}{dw_1^2} + NP_1^w \left( \frac{1 - P_1^w}{\mu^w} \right) \left( \frac{dp_1}{dw_1} - 1 \right) \left[ 2 - \left( \frac{1 - 2P_1^w}{1 - P_1^w} \right) \right] \\
&= NP_1^w \frac{d^2 p_1}{dw_1^2} + NP_1^w \left( \frac{1 - P_1^w}{\mu^w} \right) \left( \frac{dp_1}{dw_1} - 1 \right) \left( \frac{2 - 2P_1^w - 1 + 2P_1^w}{1 - P_1^w} \right) \\
&= NP_1^w \frac{d^2 p_1}{dw_1^2} + N \left( \frac{P_1^w}{\mu^w} \right) \left( \frac{dp_1}{dw_1} - 1 \right).
\end{aligned} \tag{45}$$

Now

$$\begin{aligned}
\frac{d^2 p_1}{dw_1^2} &= \frac{d}{dw_1} \left[ \frac{-\mu}{1 + (1 - \mu)\Phi} \right] \\
&= \frac{-\mu^2}{[1 + (1 - \mu)\Phi]^2} \frac{\partial \Phi}{\partial p_1} \frac{dp_1}{dw_1} \\
&= \frac{\mu^3}{[1 + (1 - \mu)\Phi]^3} \left[ \frac{\partial P_{1|1}^s}{\partial p_1} - \frac{\partial P_{1|-1}^s}{\partial p_1} \right] \\
&= \frac{\mu^3}{[1 + (1 - \mu)\Phi]^3} \left[ -\frac{1}{\mu^d} [1 - P_{1|1}^s] P_{1|1}^s + \frac{1}{\mu^d} [1 - P_{1|-1}^s] P_{1|-1}^s \right] \\
&= \frac{-\mu^3}{\mu^d [1 + (1 - \mu)\Phi]^3} \Phi [1 - P_{1|1}^s - P_{1|-1}^s].
\end{aligned} \tag{46}$$

From our model assumptions  $0 < \mu = \frac{\mu^w}{\mu^d} > 1$ . Further, we know that, at the candidate

symmetric equilibrium,  $\Phi > 0$ ,  $P_{1|1}^s = \frac{1}{1 + (n-1)\lambda}$  and  $P_{1|-1}^s = \frac{\lambda}{1 + (n-1)\lambda}$ , where

$\lambda \equiv e^{-\alpha^d t / \mu^d} > 0$ . Hence  $\left[1 - P_{1|1}^s - P_{1|-1}^s\right] = \frac{(n-2)\lambda}{1+(n-1)\lambda}$  is non-negative for  $n \geq 2$ . Thus

(46) is non-positive.

Substituting from (46) in (45) means that the first term on the right hand side of (45) is non-positive. We also know from (42) that  $dp_1/dw_1 < 0$ , so the second term in (45) is negative.

Hence  $\frac{d^2 \pi_1}{dw_1^2}$  is strictly negative at any extremum (solution of the first-order equations) and thus

the profit is quasi-concave. As a consequence, the candidate Nash equilibrium is a Nash equilibrium. QED.

### Appendix A3: Proof of Proposition 4

Recall from (26) that with trip chaining

$$CS = \mu^w \log \left[ ne^{\frac{w^* - \alpha^w t + CS^s - \beta}{\mu^w}} \right], \quad (47)$$

where  $CS^s = h - p^* + \mu^d \log[1 + (n-1)\lambda]$  and  $\lambda \equiv e^{-\alpha^d t / \mu^d}$ . By substitution, the above equation can be reformulated as

$$CS = \mu^w \log n + w^* - p^* + h - \beta - \alpha^w t + \mu^d \log[1 + (n-1)\lambda]. \quad (48)$$

Note that, for the case without trip chaining

$$CS_{1|nc}^s = CS_{-1|nc}^s = h - \alpha^d t + \mu^d \log[D_{1|nc}^s] \quad (49)$$

and

$$P_{1|nc}^w(w_1, w^*) = \exp \left( \frac{w_1 - \alpha^w t + CS_{1|nc}^s}{\mu^w} \right) D_{1|nc}^{w-1}, \quad (50)$$

where  $D_{1|nc}^w = \exp \left( (-\alpha^w t + CS_{1|nc}^s) / \mu^w \right) \left[ \exp(w_1 / \mu^w) + (n-1) \exp(w^* / \mu^w) \right]$ .

Similarly using (49) and the log sum of the denominator of (50) both evaluated at the symmetric equilibrium, we obtain

$$CS_{ntc} = \mu^w \log \left[ ne^{\frac{w_{ntc}^* - \alpha^w t + CS_{ntc}^s - \beta}{\mu^w}} \right] \quad (51)$$

for the reference case without trip chaining, where  $CS_{ntc}^s = h - p_{ntc}^* - \alpha^d t + \mu^d \log n$ .

Substituting for  $CS_{ntc}^s$  in (51) and subtracting the resulting equation from (48) leads to

$$\begin{aligned} CS - CS_{ntc} &= (p_{ntc}^* - w_{ntc}^*) - (p^* - w^*) + \alpha^d t + \mu^d \log n + \mu^d \log [1 + (n-1)\lambda] \\ &= (p_{ntc}^* - w_{ntc}^*) - (p^* - w^*) + \mu^d \log \left[ 1 + \frac{\lambda^{-1} - 1}{n} \right] \end{aligned} \quad (52).$$

QED.

#### Appendix A4: Proof of Lemma 2

Call the welfare optimum number of firms without trip chaining  $n_{ntc}^0$  and the optimal number of firms when trip chaining is possible  $n^0$ .

The optimal number of firms in the absence of trip chaining maximizes the following welfare function (per household):

$$W(n) = \Psi - \frac{n}{N} (F + K) + (\mu^d + \mu^w) \log n,$$

where  $\Psi = \theta(1 - \beta) - c^1 + \alpha^h t + h - \alpha^d t - \beta - \alpha^w t$ , so the optimal number of firms satisfies the following first order condition:

$$\frac{\mu^w}{n_{ntc}^0} + \frac{\mu^d}{n_{ntc}^0} = \frac{F + K}{N}.$$

When trip chaining is allowed, the welfare function (per household) to be maximised is

$$W(n) = \Psi' + \mu^w \log n + \mu^d \log [1 + (n-1)\lambda] - \frac{n}{N} (F + K),$$

where  $\Psi' = \Psi + \alpha^d t$ . This leads to the first order condition

$$\frac{\mu^w}{n^0} + \frac{\mu^d \lambda}{1 + (n^0 - 1)\lambda} = \frac{F + K}{N}.$$

Comparing both first order equations, the solutions must satisfy:  $n_{mc}^0 \geq n^0$ . Q.E.D.

### Appendix A5: Proof of Lemma 3

We first show that in the symmetric equilibrium with trip chaining allowed, the profit per firm is a decreasing function of the number of firms. We limit ourselves to the case where  $n \geq 2$ . Taking the derivative of the profit equations for firms when trip chaining is possible, (23), with respect to  $n$  we find:

$$\begin{aligned} \frac{\partial \pi}{\partial n} &= -\frac{(\mu^w + \mu^d)}{(n-1)^2} + (1-\mu)(1-\lambda) \\ &\quad \left[ \frac{\mu^d}{(n-1)^2(2-\mu+(n-2)\lambda+\mu\lambda)} + \frac{\mu^d}{(n-1)} \frac{\lambda}{(2-\mu+(n-2)\lambda+\mu\lambda)^2} \right] \\ \text{Thus } \text{sign} \frac{\partial \pi}{\partial n} &= \text{sign} \left\{ -(\mu^w + \mu^d) + \left[ \mu^d + \frac{\lambda(n-1)\mu^d}{(2-\mu+(n-2)\lambda+\mu\lambda)} \right] \frac{(1-\mu)(1-\lambda)}{2-\mu+(n-2)\lambda+\mu\lambda} \right\} \end{aligned}$$

The first term on the right hand side is negative. Because  $\mu$  and  $\lambda$  are both smaller than one, we know that the term  $\frac{(1-\mu)(1-\lambda)}{2-\mu+(n-2)\lambda+\mu\lambda}$  is at most equal to one, we will use therefore the upper bound for this term and put this term equal to one.

It is therefore sufficient to show that the following expression is negative:

$$\begin{aligned} &\text{sign} \left\{ -\mu^w [2-\mu+(n-2)\lambda+\mu\lambda] + \lambda(n-1)\mu^d \right\} \\ &= \text{sign} \left\{ -2\mu^w + \mu^d - \mu^w(n-2)\lambda - \mu^d\lambda + \lambda(n-1)\mu^d \right\}, \\ &= \text{sign} \left\{ -\mu^w + (\mu^d - \mu^w) + (n-2)\lambda(\mu^d - \mu^w) \right\} \end{aligned}$$

where we have used the definition  $\mu \equiv \mu^w / \mu^d > 1$  and this last expression is indeed negative.

The optimal number of firms can be found by using the first order condition for a maximum of the welfare function



$$\frac{\mu^w}{n^0} + \frac{\mu^d \lambda}{1 + (n^0 - 1)\lambda} - \frac{F + K}{N} = 0 \quad (53)$$

and the equation that determines the free entry equilibrium number of firms is (using (23) and the zero profit condition)

$$\frac{(\mu^w + \mu^d)}{n - 1} - \frac{\mu^d}{n - 1} \left[ \frac{(1 - \mu)(1 - \lambda)}{(2 - \mu + (n - 2)\lambda + \mu\lambda)} \right] - \frac{F + S}{N} = 0. \quad (54)$$

We know from the above that the left hand side of (54) is a decreasing function of  $n$ . Moreover we know that the profit goes to infinity (or at least a very large number) when  $n$  approaches one. The left hand side (LHS) of (53) is however finite when  $n$  approaches one. This means that starting from a value of  $n=1$ , the LHS of (53) is initially always smaller than the LHS of (54). Since both LHS are decreasing, it is sufficient to prove that the LHS of (53) and (54) can never be equal to know that the solution of (54) is always larger than the solution of (53).

We now show that there is no value of  $n > 0$  that satisfies the LHS of both equations. Equating the left hand sides of (53) and (54) and rearranging, we have

$$\frac{\mu^w}{n} = \mu^d \left\{ \frac{-1}{1 + (n - 1)\lambda} + \frac{(1 - \mu)(1 - \lambda)}{(2 - \mu + (n - 2)\lambda + \mu\lambda)} \right\},$$

which can be rewritten as

$$\frac{\mu^w}{n} = \mu^d \left\{ \frac{-[2 - \mu + (n - 2)\lambda + \mu\lambda] + (1 - \mu)(1 - \lambda)[1 + (n - 1)\lambda]}{[1 + (n - 1)\lambda][2 - \mu + (n - 2)\lambda + \mu\lambda]} \right\}. \quad (55)$$

The LHS is always positive and the denominator of the RHS is always positive in (55), so it is sufficient to prove that the numerator is always negative to prove our result. The numerator of the LHS of (55) can be simplified to  $-1 - \lambda(n - 1)\{\mu - \lambda\mu + \lambda\}$ . This is always negative given that  $\mu$  and  $\lambda$  are both smaller than one. Q.E.D.

## REFERENCES

Anderson, S.P., A. de Palma, and J.F. Thisse, 1989, "Demand for differentiated products, discrete choice models, and the characteristics approach," *Review of Economic Studies*, 56, 21-35.

Anderson, S.P., A. de Palma, and J.F. Thisse, 1992, *Discrete choice theory of product differentiation*,. Cambridge, MA: The MIT press.

Anderson, S.P. and A. de Palma, 2001, "Product Diversity in Asymmetric Oligopoly – Is the quality of consumer goods too low?" *Journal of Industrial Economics*, 2, 113-135.

Ben-Akiva, M.E. and S. Lerman, 1979, "Disaggregate travel and mobility choice models and measures of accessibility," D. Hensher and P.R. Stopher, eds, *Behavioral travel modelling*, Croon Helm, London.

Ben-Akiva, M.E. and S. Lerman, 1985, *Discrete Choice analysis – Theory and Application to travel demand*, Cambridge, MA, MIT press.

Bhat, C.R., Frusti, T., Zhao, H., Schonfelder, S. and K.W. Axhausen, 2004a, "Intershopping Duration: An Analysis Using Multiweek Data," *Transportation Research Part B*, 38, 39-60.

Bhat, C.R., J.Y. Guo, S. Srinivasan and A. Sivakumar, 2004b, "A Comprehensive Econometric Microsimulator for Daily Activity-Travel Patterns," *Transportation Research Record: Journal of the Transportation Research Board*, 1894, 57-66.

Bhat, C.R., 2008, "The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions," Forthcoming, *Transportation Research Part B*.

Claycome, R.J., 1991, "Spatial retail markets," *International Journal of Industrial Organization*, 9, 303-313.

Claycombe, R.J. and T.E. Mahan, 1993, "Spatial aspects of retail market structure – beef pricing revisited," *International Journal of Industrial Organization*, 11, 283-291.

de Palma, A. and S. Proost, 2006, "Imperfect competition and congestion in the city," *Journal of Urban Economics*, 60, 185-209.

Eaton, B.C. and R.G. Lipsey, 1982, "An economic theory of central places," *Economic Journal*, 92, 56-72.

Fujita M. and J.F. Thisse, 2002, *Economics of agglomeration, cities, industrial location and regional growth*, Cambridge University Press.

Golob, T.F. and D.A. Hensher, 2007, "The trip chaining activity of Sydney residents: A cross-section assessment by age group with a focus on seniors," *Journal of Transport Geography*, 15, 298-312.

Hensher, D.A. and A.J. Reyes, 2000, "Trip Chaining as a Barrier to the Propensity to Use Public Transport," *Transportation*, 27, 341-361.

McFadden, D., 1978, "Modelling the choice of residential location," A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull, eds, *Spatial interaction theory and planning models*, North-Holland, Amsterdam, 75-96.

McGuckin, N., Zmud, J. and Y. Nakamoto, 2005, "Trip chaining trends in the US – Understanding travel behavior for policy making," *Transportation Research Record*, 1917, 99-204.

Raith, M., 1996, "Spatial retail markets with commuting consumers," *International Journal of Industrial Organisation*, 14, 447-463.

Rouwendal, J. and P. Rietveld, 1999, "Prices and opening hours in the retail sector: Welfare effects of restrictions on opening hours," *Environment and Planning A*, 31, 2003-2016.

Thill, J.-C. and I. Thomas, 1987, "Towards conceptualising trip-chaining behavior: A review," *Geographical Analysis*, 19, 1-17.

Thisse, J-F. and X. Vives, 1988, "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78, p 122-137.

FIGURES

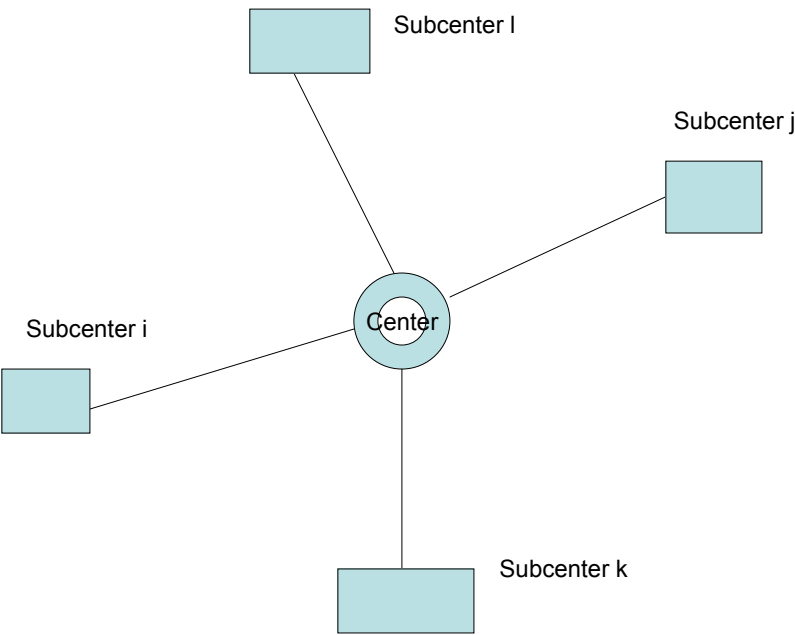


Figure 1

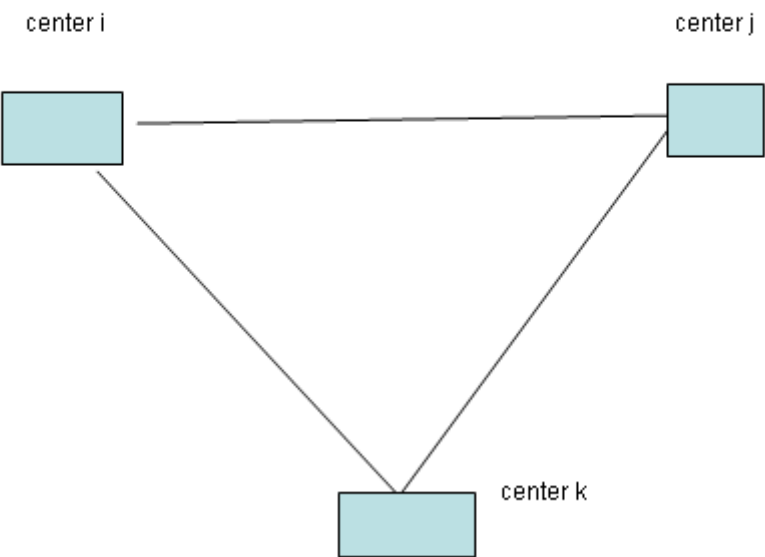


Figure 2

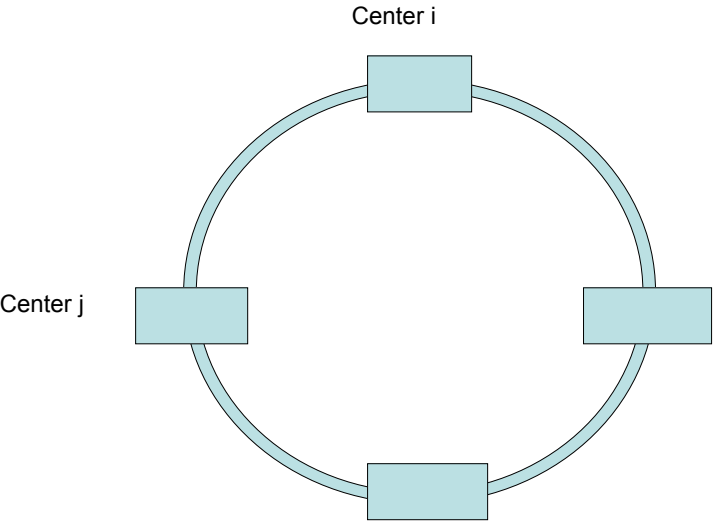


Figure 3

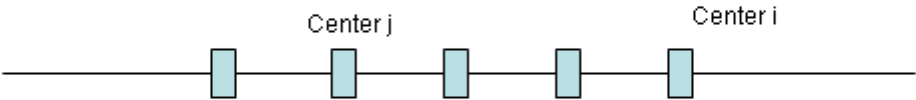


Figure 4

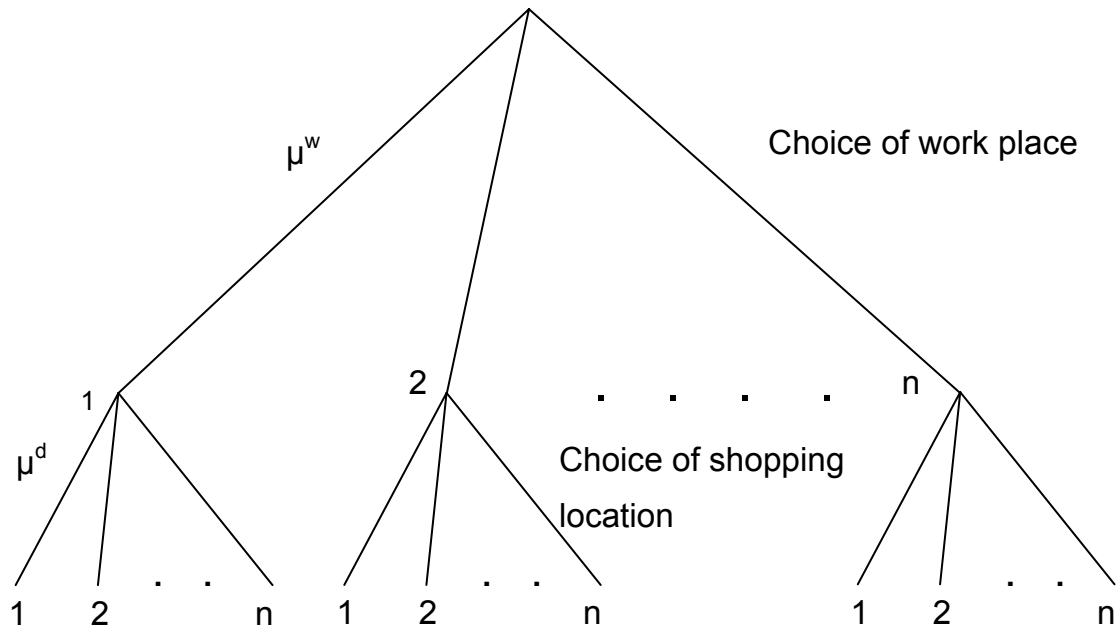


Figure 5

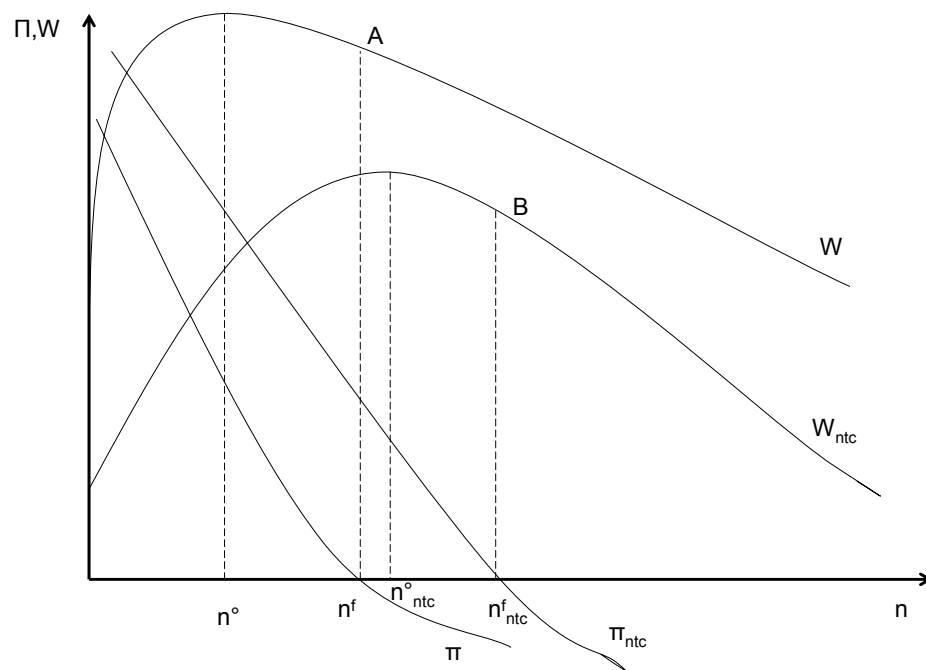


Figure 6